Approximation Method for Configuration Optimization of Trusses

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An efficient method for truss configuration optimization is presented. Elastic trusses are designed for minimum weight by varying the areas of the members and the location of the joints. Constraints on member stresses and Euler buckling are imposed, and multiple loading conditions are considered. The method presented here utilizes an approximate structural analysis based on first-order Taylor series expansions of the member forces. The force approximation succeeds in reducing the degree of coupling between the sizing and geometry design variables and thereby increases the accuracy of the approximation. A numerical optimizer uses information from the approximate structural analysis as it minimizes the weight of the truss.

Introduction

OPTIMIZING the geometry of trusses has been a research topic in structural optimization for some time. However, none of the methods proposed thus far have been efficient or reliable enough to put into widespread use. The purpose of this paper is to present an efficient method of truss configuration optimization that outperforms previous methods by an order of magnitude in terms of the number of structural analyses required to find an optimum. Although the scope of this research is limited to trusses, it should be noted that Kodiyalam and Vanderplaats[1] have recently used a similar method for shape optimization for three-dimensional continuum structures.

Vanderplaats and Moses[2] optimized a variety of indeterminate trusses by alternating the sizing and configuration design subspaces during the optimization process. Different methods were applied to each of the subsets of variables in order to take advantage of the behavior of each design space. The sizing problem was solved with a fully stressed design method, and the configuration problem was solved with a constrained steepest descent method. The method proved capable of designing large-scale structures. Pederson[3] independently used a similar approach.

Imai[4] used an Augmented Lagrange multiplier method that treated both sizing and configuration variables together in the same design space. The method was able to handle initially infeasible designs, and approximations were used during the one-dimensional search.

Schmit and Miura[5] introduced the idea of creating an approximate structural analysis based on Taylor expansions of various structural responses. Although this approach was initially used for sizing problems only, it proved to be especially efficient. Vanderplaats[6] incorporated these concepts into the two-subspace approach for efficient configuration optimization that included displacement constraints. Felix and Vanderplaats[7] later added constraints on the natural frequency.

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A variety of other significant works have been published in the field of truss configuration optimization. For reviews of the literature, see Vanderplaats[8] and Topping.[9]

The method presented here expands on work done by Vanderplaats and Salajegheh[10] for sizing problems. Their approach was to form an approximate structural analysis which used a Taylor series expansion of the member forces. During the approximate optimization, these forces were divided by the member areas to estimate the member stresses. This approach proved to be more efficient than the method of directly linearizing the stresses. An even greater increase in efficiency, as compared to previous methods, is achieved here by including configuration variables into the formulation.

Truss Design Problem

The objective function for the truss configuration problem is the total weight of the structure. This is represented as

$$W = \sum_{i=1}^{NM} \rho_i A_i L_i \tag{1}$$

where i is an individual member, NM is the total number of members, ρ_i is the material density, and A_i and L_i are the cross-sectional areas and length of the ith member, respectively.

The following constraints on structural behavior must be satisfied

a) Member stresses

$$\sigma_i^- \leq \sigma_{ij} \leq \sigma_i^+$$

where σ_i^- is the maximum compressive stress and σ_i^+ is the maximum tensile stress for member i under load condition j.

b) Euler buckling

$$\sigma_{bi} \leq \sigma_{ij}$$

σ_{bi} is the Euler buckling compressive stress limit for truss member i. It is taken as

$$\sigma_{bi} = \frac{-K_i E_i A_i}{L^2_i}$$

where K_i is a constant determined from the cross-sectional geometry and E_i is the Young's Modulus of the material. Local crippling and system buckling constraints are not considered in this study.

c) Limits on areas

$$A_i^{min} \leq A_i \leq A_i^{max}$$

where A_i^{min} and A_i^{max} are the minimum and maximum member areas. For this study, the range of allowable areas is the same for every member.

d) Linking of areas

A practical consideration in design is to limit the number of members with unique cross-sectional areas. Hence, member areas may be linked in the design process as

$$A_k = A_i$$

where A_k is a dependent and A_i is an independent area variable.

d) Limits on coordinate variables

Coordinates may be constrained during the optimization as follows

$$X_j^- \leq X_j \leq X_j^+$$

where X_j^- and X_j^+ are lower and upper bounds on the location of joint j.

e) Linking of coordinate variables

Coordinate variables are often linked to preserve the symmetry of a structure. The relationship between linked coordinates is

$$X_k = a_k + b_k X_i$$

where a_k and b_k are constants; X_k is a dependent coordinate variable; and X_i is an independent coordinate variable.

The numerical optimizers used for this study are the ADS[11] and DOT[12] programs.

Structural Analysis

Two different methods of structural analysis are used to evaluate the constraints: finite element and approximate. The former is the finite element displacement method, which is fairly expensive in terms of computer time. In order to reduce the number of finite element analysis during the optimization process, an approximate structural analysis is created based on Taylor series expansions of member forces. The approximate analysis, though not as accurate as the actual analysis, is less costly in terms of computer time.

Force vs Stress Approximations

The approximate structural analysis is based on a Taylor series expansion of the member forces. The advantage of choosing an approximation of forces rather than a direct stress approximation can be seen in the following example. A one-bar determinate truss is shown in Fig. 1. If the X-coordinate of node 2 and the cross-sectional area of the member are to be taken as variables, the Taylor series expansions of the stress and force of the member can be written as follows:

$$\sigma = \sigma^0 + \frac{\partial \sigma}{\partial A} \cdot \delta A + \frac{\partial \sigma}{\partial X} \cdot \delta X \tag{2}$$

where

$$\frac{\partial \sigma}{\partial A} = \frac{-P\sqrt{H^2 + X^2}}{HA^2} \tag{3a}$$

$$\frac{\partial \sigma}{\partial X} = \frac{PX}{2AH\sqrt{H^2 + X^2}} \tag{3b}$$

and

$$F = F^0 + \frac{\partial F}{\partial A} \cdot \delta A + \frac{\partial F}{\partial X} \cdot \delta X \tag{4}$$

where

$$\frac{\partial F}{\partial A} = 0 \tag{5}$$

$$\frac{\partial F}{\partial X} = \frac{PX}{H\sqrt{H^2 + X^2}} \tag{6}$$

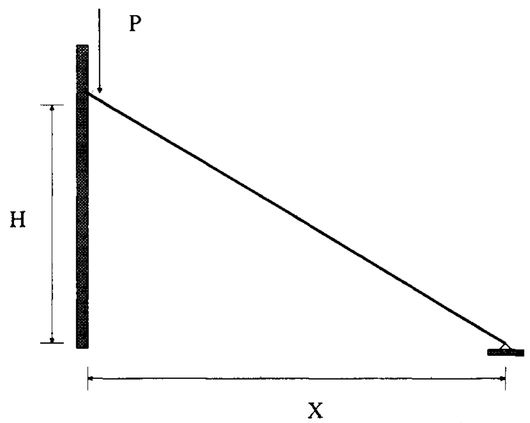

Fig. 1 Determinate 1-bar truss.

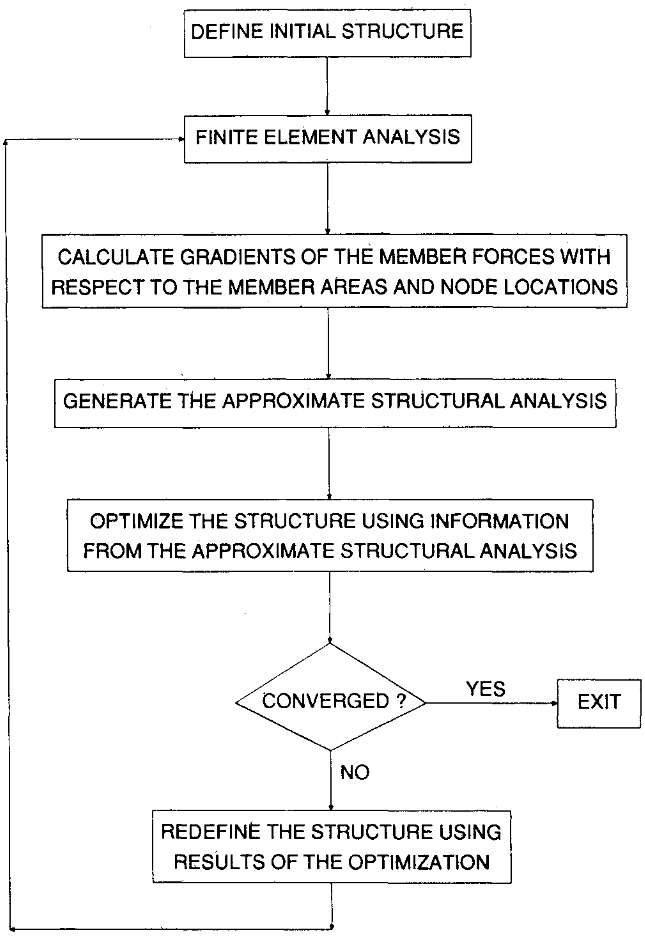

Fig. 2 Algorithm for truss configuration optimization.

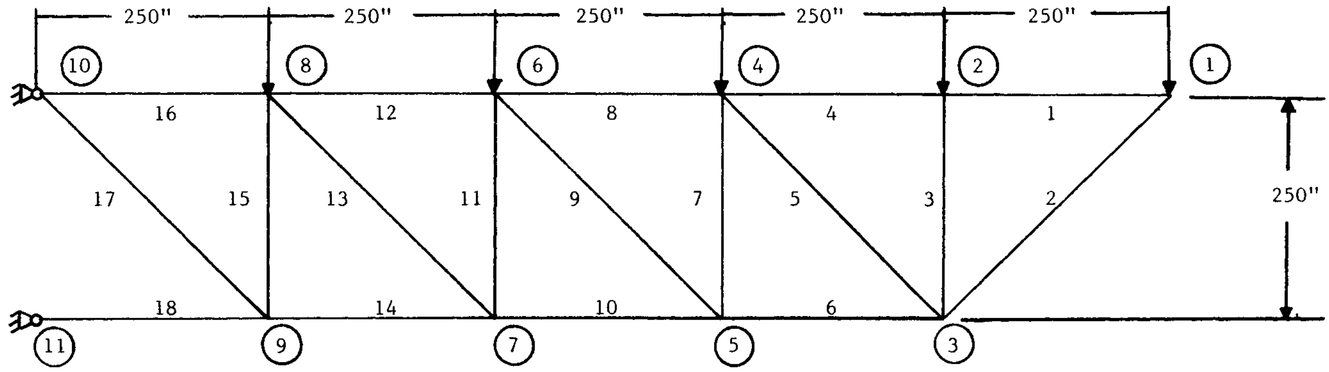

Fig. 3 18-bar truss, initial geometry.

In the above formulation,

$$\delta A = A - A^0$$

$$\delta X = X - X^0$$

In the direct stress expansion, the geometry and sizing terms are coupled to each other, the member area appears in both denominators, and the coordinate X appears in both numerators. In the force approximation, however, the sizing and configuration terms are decoupled. This decoupling of the sizing and configuration terms is significant because the optimizer is able to change the member area without degrading the quality of the configuration component of the approximation. Some coupling will still exist in an indeterminate structure, but it is reduced by using the force approximation.

Approximate Optimization Problem

The approximate optimization problem is formally expressed as follows.

Minimize $W = \sum_{i=1}^{NM} \rho_i A_i L_i$

Subject to

$$\frac{F_{ij}/A_i}{\sigma_i^+} - 1.0 \leq 0.0, \qquad i = 1, NM, \qquad j = 1, NLC$$

$$1.0 - \frac{F_{ij}/A_i}{\sigma_i^-} \leq 0.0, \qquad i = 1, NM, \qquad j = 1, NLC$$

$$1.0 - \frac{F_{ij}/A_i}{\sigma_{bi}} \leq 0.0, \qquad i = 1, NM, \qquad j = 1, NLC$$

$$A_i^- \leq A_i \leq A_i^+ \qquad i = 1, NM$$

$$X_k^- \leq X_k \leq X_k^+ \qquad K = 1, NJ$$

where NM is the number of members, NLC is the number of loading conditions, NJ is the number of joints, σ_i^+ is the tensile allowable stress for the ith member, σ_i^- is the compressive allowable stress for the ith member, and σ_{bi} is the Euler buckling compressive stress limit for member i. The third constraint is used only when Euler buckling constraints are imposed. The approximate forces F_{ij} are the Taylor series expansions for the ith member force under load condition j. The Taylor series expansion for the force in member i under load condition j is expressed as

$$\tilde{F}_{ij}(A,X) = F_{ij}(A^0,X^0) + \sum_{k=1}^{NM} \frac{\partial F_{ij}}{\partial A_k} \cdot (A_k - A_k^0) + \sum_{L=1}^{NM} \frac{\partial F_{ij}}{\partial X_L} \cdot (X_L - X_L^0) \qquad (7)$$

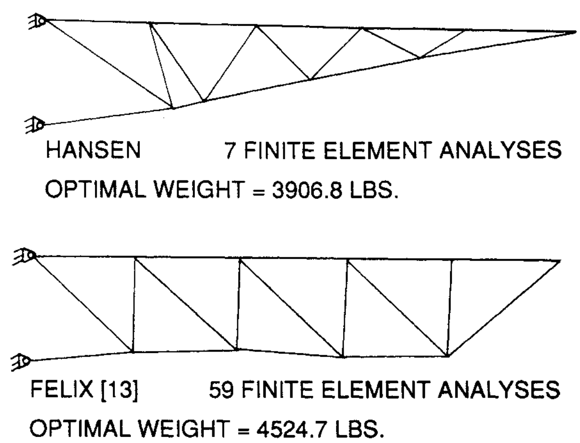

Fig. 4 Comparison between initial and final geometries of the 18-bar planar truss with stress constraints.

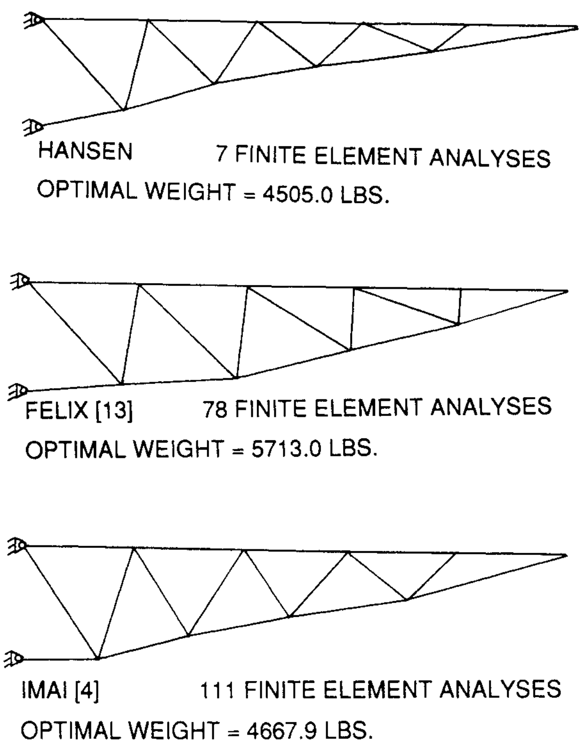

Fig. 5 Comparison between initial and final geometries for the 18-bar planar truss with stress and Euler buckling constraints.

Table 1 8-bar planar truss

Joint	Load (lbs) F_x	F_y
1	0.0	− 20,000.0
2	0.0	− 20,000.0
4	0.0	− 20,000.0
6	0.0	− 20,000.0
8	0.0	− 20,000.0

Constants
Young's modulus = 1.0×10^7 psi
Allowable stress = 20,000.0 psi
Density = 0.1 lb/in.^{3}
Buckling coefficient = 4.0

Table 2 18-bar planar truss, design information

Variable	Initial value	Final value	Ref. 13
Constraints on stress			
A_1	10.00	10.71	11.05
A_2	15.00	15.19	15.07
A_3	1.00	1.94	4.54
A_4	7.07	5.19	5.33
X_3	1000.0	881.4	991.2
Y_3	0.0	178.8	19.7
X_5	750.0	628.9	745.9
Y_5	0.0	124.9	15.1
X_7	500.0	390.5	494.6
Y_7	70.0	66.8	34.5
X_9	250.0	313.2	249.5
Y_9	0.0	45.0	23.6
Weight	4780.5	3906.8	4524.7
Numer of analyses		8	59

Table 3 18-bar planar truss, design information

Variable[a]	Initial	Final	Ref. 13	Ref. 4
Constraints on stress and Euler buckling				
A_1	10.00	12.76	11.34	11.24
A_2	21.65	17.77	19.28	15.68
A_3	12.50	5.55	10.97	7.93
A_4	7.07	3.26	5.30	6.49
X_3	1000.0	881.4	994.6	891.1
Y_3	0.0	178.8	162.3	143.6
X_5	750.0	628.9	747.4	608.2
Y_5	0.0	124.9	102.9	105.4
X_7	500.0	390.5	482.9	381.7
Y_7	0.0	66.8	33.0	57.1
X_9	250.0	313.2	221.7	181.0
Y_9	0.0	45.0	17.1	− 3.2
Weight	6430.7	4505.0	5713.0	4667.9
Number of analyses		8	78	111

[a]Areas are in in.^{2}, coordinates are in inches.

Table 4 25-bar space tower

Load conditions and constants

Joint	F_x	F_y	F_z
Load condition 1 (lbs)			
1	0.0	20,000.0	− 5000.0
2	0.0	− 20,000.0	− 5000.0
3	0.0	0.0	0.0
6	0.0	0.0	0.0
Load condition 1 (lbs)			
1	1000.0	10,000.0	− 5000.0
2	0.0	− 10,000.0	− 5000.0
3	500.0	0.0	0.0
6	500.0	0.0	0.0

Constants
Young modulus = 1.0×10^7 psi
Allowable stress = 20,000.0 psi
Density = 0.1 lb/in.^{3}
Buckling coefficient = 4.0

Table 5 25-bar space tower

Var.	Initial	Final	Second initial	Final[a]	Ref. 6	Ref. 13
Constraints on stress and Euler buckling						
A_1	0.009	0.010	10.0	0.491	0.032	0.013
A_2	0.782	0.487	10.0	0.633	0.565	0.414
A_6	0.754	0.836	10.0	0.894	0.811	0.842
A_{10}	0.001	0.021	10.0	0.242	0.028	0.033
A_{12}	0.130	0.123	10.0	1.261	0.047	0.101
A_{14}	0.558	0.084	10.0	0.184	0.097	0.121
A_{18}	0.982	0.698	10.0	0.752	0.749	0.739
A_{22}	0.801	0.548	10.0	0.514	0.551	0.554
X_4	37.5	23.7	37.5	6.6	12.9	21.5
Y_4	37.5	49.3	37.5	44.0	48.2	48.3
Z_4	100.0	97.7	100.0	90.2	97.4	100.3
X_8	100.0	27.5	100.0	26.4	37.1	22.1
Y_8	100.0	96.4	100.0	80.1	94.1	96.4
Weight (lbs)	229.9	128.3	3307.2	149.7	133.5	128.5
No. of analyses		7		7	171	N/A

[a]This optimum is obtained without altering any default parameters in the optimizer. That is, in this case the optimizer is used as a "black box" without being fine-tuned. This, along with the fact that the initial areas are all set to 10.0 in.^{2}, accounts for the premature convergence to a greatly improved, although nonoptimum, design.

Table 6 47-bar planar tower

Joint	F_x	F_y
Load condition 1 (lbs)		
17	6000.0	− 14,000.0
22	6000.0	− 14,000.0
Load condition 2 (lbs)		
17	6000.0	− 14,000.0
22	0.0	0.0
Load condition 3 (lbs)		
17	0.0	0.0
22	6000.0	− 14,000.0

Constants
Young's modulus = 3.0×10^7 psi
Allowable tensile stress = 20,000.0 psi
Allowable compressive stress = − 15,000.0 psi
Density = 0.3 lb/in.^{3}
Buckling coefficient = 3.96

Table 7 47-bar planar tower, sizing design information

Member[a]	Initial	Final	Ref. 13	Ref.2
Constraints on stress and Euler buckling				
3	3.80	2.42	2.73	2.80
4	3.40	2.35	2.47	2.56
5	0.80	0.82	0.73	0.77
7	0.90	0.10	0.21	10^{-6}
8	0.90	0.86	0.94	0.67
10	1.80	1.15	1.08	1.80
12	2.10	1.77	1.69	1.94
14	1.20	0.67	0.69	0.65
15	1.60	0.86	1.06	1.06
18	2.10	1.24	1.41	1.695
20	0.70	0.33	0.26	0.16
22	0.90	1.22	0.81	0.61
24	1.70	0.93	1.06	1.31
26	1.70	0.86	1.05	1.32
27	1.40	0.69	0.82	1.05
28	0.90	0.15	0.30	0.52
30	3.70	2.46	2.77	2.94
31	1.50	0.90	0.66	0.73
33	0.70	0.10	0.21	10^{-6}
35	2.90	2.74	2.90	3.16
36	0.70	0.92	0.27	0.97
38	1.60	0.10	1.41	0.18
40	3.70	2.94	3.43	3.47
41	1.60	1.13	0.99	1.02
43	0.70	0.10	0.17	10^{-6}
45	4.50	3.12	3.65	3.71
46	1.60	1.10	1.01	0.93

[a]Areas are in inches.^{2}

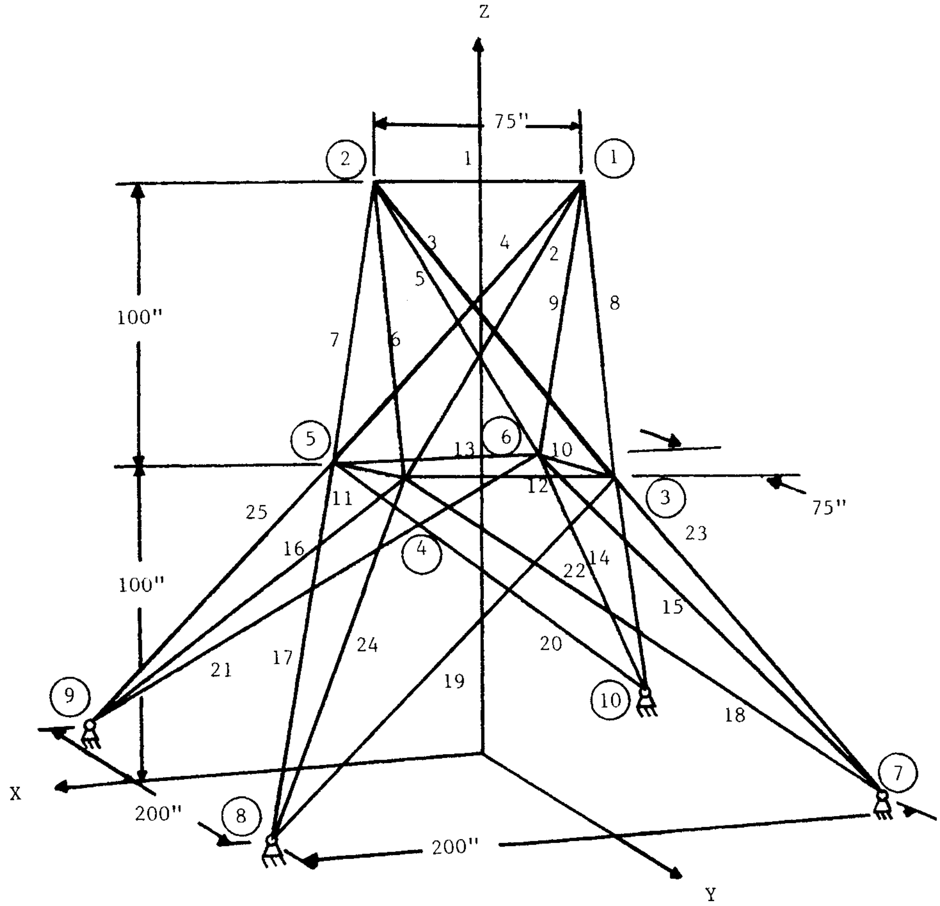

Fig. 6 25-bar space truss, initial geometry.

This approximation is assumed to be valid in the neighborhood about the point that the approximation is taken.

Algorithm

The overall process of truss configuration optimization incorporates the analysis, approximation, and optimization methods that have been discussed above. A block diagram of the process is presented in Fig. 2.

Numerical Examples

Some standard test cases that have been used in previous papers on truss configuration optimization were run using the computer program developed for this study. These cases include an 18-bar planar truss, a 25-bar space truss subjected to two load conditions, and a 47-bar planar tower subjected to three load conditions. Comparisons are made between the number of finite element structural analyses required by previous methods and the number required by the present method.

Case 1: 18-Bar Planar Truss with Stress Constraints

The first example is the 18-bar planar truss previously used as a test case in Ref. 13. The initial geometry is shown in Fig. 3. The truss is subject to a single load condition and simple stress constraints with $\sigma^{\pm} = \pm 20{,}000$ psi. The element areas are linked as follows: $A1 = A4 = A8 = A12 = A16$; $A2 = A6 = A10 = A14 = A18$; $A3 = A7 = A11 = A15$; $A5 = A9 = A13 = A17$. The geometry variables are the following coordinates: $X3$, $Y3$, $X5$, $Y5$, $X7$, $Y7$, $X9$, $Y9$. There are a total of eight independent coordinate variables and four independent area variables.

The applied loads and material constants are given in Table 1. The data for the initial and final designs is given in Table 2. The design obtained for the truss subject to stress constraints only is shown in Fig. 4. Eight finite element analyses are required to obtain the final design. This compares with the 59 analyses that were required in Ref. 13.

Case 2: 18-Bar Planar Truss with Stress and Euler Buckling Constraints

In this case an additional constraint on Euler buckling is added. The buckling constant is taken to be $K = 4$. The initial

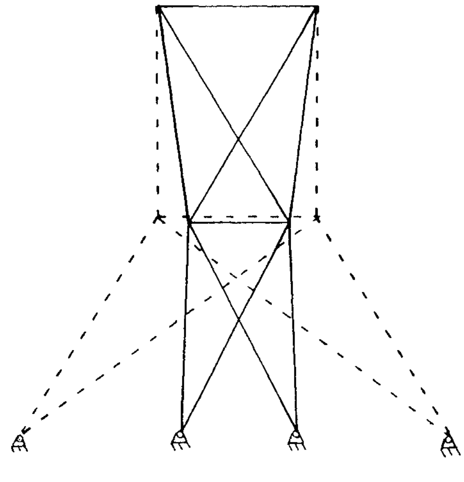

Fig. 7 Comparison of initial and optimal designs for the 25-bar space truss with stress and Euler buckling constraints, X-Z plane.

and final design data is given in Table 3. The design obtained for the truss subject to stress and Euler buckling constraints is shown in Fig. 5 compared with previous results. Eight finite element analyses are required to obtain the final design. This compares with 78 analyses required in Ref. 13 and 111 analyses required Ref. 4.

Case 3: 25-Bar Space Truss

The 25-bar truss shown in Fig. 6 is designed to support two independent load conditions. One load is a twisting load, and the other is an overturning load. Stress and Euler buckling constraints are imposed with an allowable stress of $\sigma^{\pm} = \pm 40{,}000$ psi and an Euler buckling constant $K = 39.274$. The load conditions and material constants are given in Table 4. The member areas are linked as follows: $A2 = A3 = A4 = A5$, $A6 = A7 = A8 = A9$, $A10 = A = 11$, $A12 = A13$, $A14 = A15 = A16 = A17$, $A18 = A19 = A20 = A21$, and $A22 = A23 = A24 = A25$. The coordinate variables are also linked to maintain symmetry. The independent geometry variables are the following coordinates: $X4$, $Y4$, $Z4$, $X8$, $Y8$.

The initial and final design data is given in Table 5. An X–Z plane projection of the final design is shown in Fig. 7. A Y–Z plane projection is shown in Fig. 8. A plot of weight vs finite element analysis number is given in Fig. 9. Seven finite element analyses were required to obtain the final design. This compares to 171 finite element analyses previously required in Ref. 6 for the same case.

Case 4: 47-Bar Planar Tower

The 47-bar planar tower shown in Fig. 10 is designed for the three separate load conditions given in Table 6. The first load condition represents the load imposed by two power lines coming into the tower at an angle. The second and third load condtions represent the cases when one of the two lines has snapped. Stress and Euler buckling constraints are imposed with a tensile allowable stress of $\sigma^{+} = 20{,}000$ psi, a compressive allowable stress of $\sigma^{-} = -15{,}000$ psi, and an Euler buckling constant $K = 3.96$. The member areas are linked as follows; $A1 = A3$, $A2 = A4$, $A6 = A5$, $A9 = A8$, $A11 = A12$, $A13 = A14$, $A16 = A15$, $A17 = A18$, $A19 = A20$, $A21 = A22$, $A23 = A24$, $A25 = A26$, $A29 = A30$, $A32 = A31$, $A34 = A35$, $A37 = A36$, $A39 = A40$, $A42 = A41$, $A44 = A45$, $A47 = A46$. The independent geometry variables are

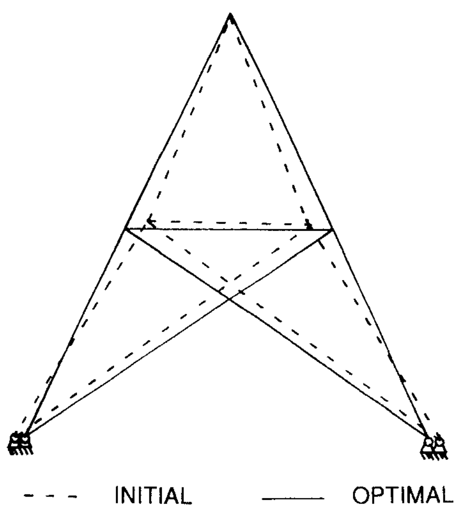

Fig. 8 Comparison of initial and optimal designs for the 25-bar space truss with stress and euler buckling constraints, *Y-Z* plane.

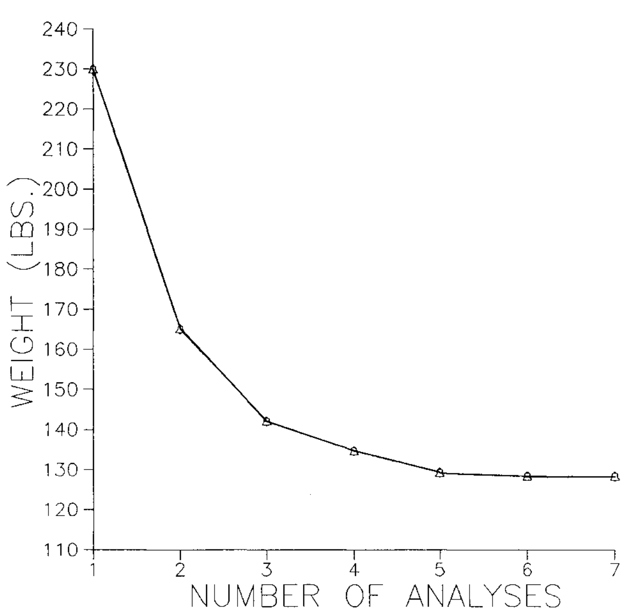

Fig. 9 Iteration history for the 25-bar space truss.

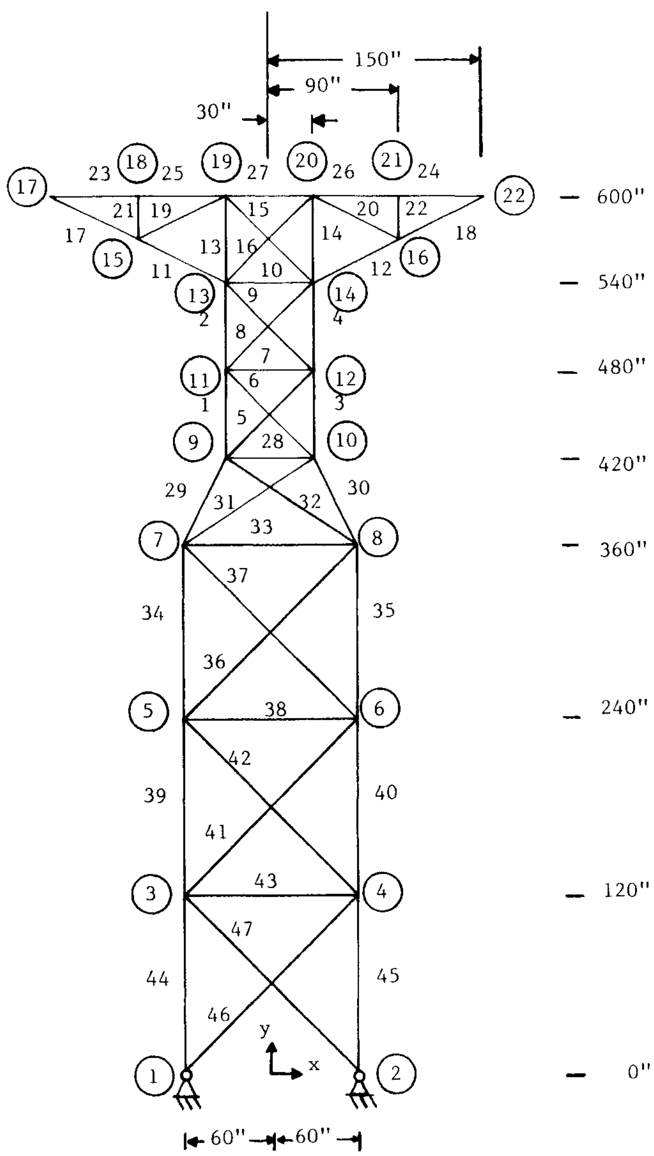

Fig. 10 47-bar planar tower, initial geometry.

Table 8 47-bar planar tower, geometry design information

Variable	Constraints on stress and Euler buckling Initial	Final	Ref. 13	Ref. 2
X_2	60.0	107.1	90.0	85.0
X_4	60.0	91.2	90.0	72.4
Y_4	120.0	122.8	123.4	121.3
X_6	60.0	74.2	83.4	59.1
Y_6	240.0	241.4	244.5	241.4
X_8	60.0	65.5	70.5	49.5
Y_8	360.0	324.6	355.1	356.8
X_{10}	30.0	57.1	60.0	42.0
Y_{10}	420.0	400.4	425.0	422.8
X_{12}	30.0	49.3	58.2	45.6
Y_{12}	480.0	472.3	478.0	480.5
X_{14}	30.0	47.4	59.6	40.2
Y_{14}	540.0	507.5	519.5	530.8
X_{20}	30.0	3.9	15.0	25.5
Y_{20}	600.0	586.5	607.6	596.5
X_{21}	90.0	83.3	96.9	90.2
Y_{21}	600.0	636.0	633.7	609.0
Weight	2445.2	1850.4	1904.0	1879.0
Number of analyses		11	N/A	N/A

[a]Coordinates are in inches.

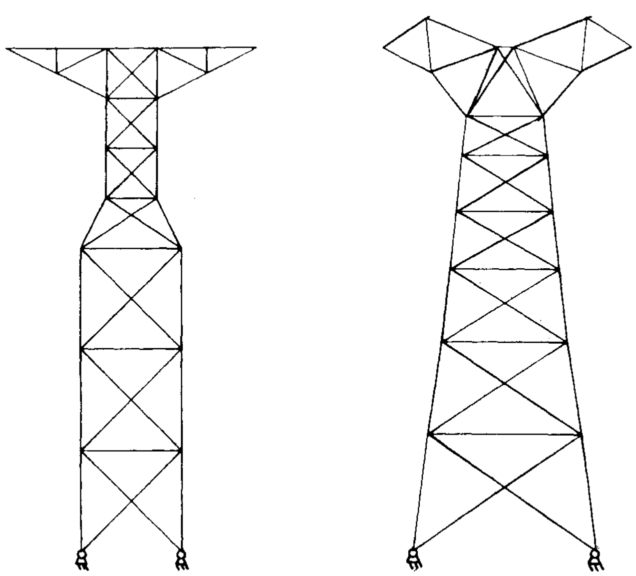

Fig. 11 Comparison between initial and optimal geometries for the 47-bar planar tower with stress and Euler buckling constraints.

$X2$, $X4$, $Y4$, $X6$, $Y6$, X8, $Y8$, $X10$, $Y10$, $X12$, $Y12$, $X14$, $Y14$, $X20$, $Y20$, $X21$, $Y21$. The geometry variables are linked to maintain symmetry about the Y-axis. Nodes 1 and 2 are required to remain at $y = 0$ and the coordinates of nodes 15, 16, 17, and 22 are not changed. There are a total of 27 independent member-sizing variables and 17 independent coordinate variables.

The initial and final design data is given in Tables 7 and 8. A comparison between the initial and optimal geometries is shown in Fig. 11. A plot of weight vs finite element analysis number is given in Fig. 12. Eleven finite element analyses were required to obtain the final design. The number of analyses required for this example in Ref. 2 is not given, but the principal author remembers it to be considerably more than 100.

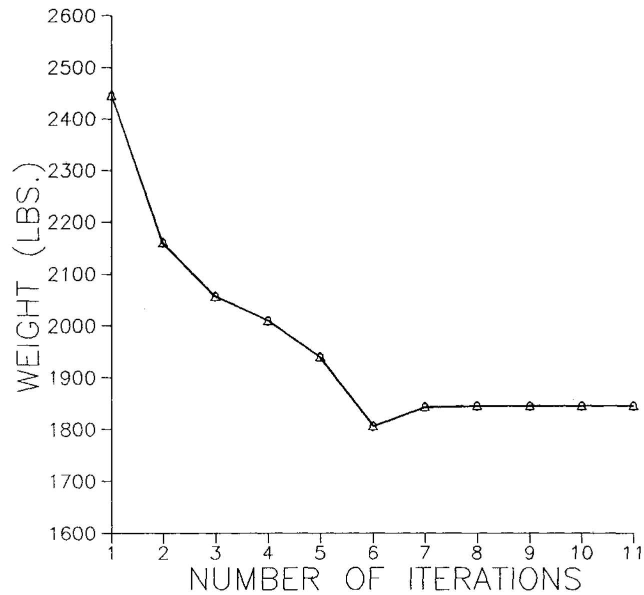

Fig. 12 Iteration history for the 47-bar tower.

Discussion

A general procedure based on force approximations has been presented for the highly efficient design of elastic trusses for optimum geometry. Two- and three-dimensional trusses subject to stress and Euler buckling constraints and multiple load conditions have been considered. The method considered here has been based on an approximate structural analysis using a Taylor-series expansion of member forces.

The first-order Taylor series expansion of member forces with respect to member areas and nodal coordinates has proven to be an accurate approximation to the true forces. It has been shown that sizing variables and geometry variables can be considered simultaneously when using this method. Furthermore, the method has been efficient and reliable for the test cases considered. The rapid rate of convergence of this method with very few finite element analyses represents a step forward in optimal truss design leading to efficiencies comparable to previous methods for member sizing alone. A "practical" optimum is typically achieved after only five detailed finite element analyses.

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